The Theory of Doubling Up

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Many authors have written about the value of not going bust in a tournament, but make no effort to quantify this effect. As a result, many players will pass up even large edges, believing (almost certainly wrongly) that even better edges will present themselves. But if this were actually true, then these players would have gigantic edges over the field in terms of equity. Observation does not bear this out. Nonetheless there *is* an effect—as usual, we seek to measure and quantify the effect, and use it as a guide in making accurate decisions at the table.

To attempt to assess the value of different tournament stacks, we can use a variety of different models. Some of these models are useful in different stages of the tournament; for example, Landrum-Burns is primarily useful when the players are near or in the money and the question is how to allocate the prizes for places other than first. But this model is not intended to be applied in the opening and middle stages of the tournament.

Yet perhaps the most misunderstood and misapplied tournament strategy concepts belong to the early and middle stages of the tournament.
The question that is perhaps the most asked and argued about with regard to no-limit tournaments boils down to the following:

How much of an edge should you pass up in the early or middle going because of your skill?

For the purposes of this discussion we will ignore the value of your time and assume that the goal is simply to increase

your equity in the tournament.
We begin, as all models do, with a few assumptions.

Assumption 1:
The chance of doubling a certain player's chip stack before that player busts is relatively constant throughout the tournament.

This assumption obviously breaks down at extremes, such as when you are all-in on the blind and the like. But these situations are not the ones that this model is designed to cover. Obviously, if the blinds are smaller, there is less variance, and so the skilled player might have a higher chance of doubling up once, but it is unclear that this chance shrinks quickly as the blinds get higher and we believe this effect to be small, if it exists at all.

Assumption 2:
We are still far enough away from the money that "chance of winning the tournament" is still a reasonable proxy for your equity in the tournament.

Most tournaments concentrate a fairly high percentage of money in the top few spots, so as long as we are fairly far from the money (more than twice the number of players who are paid are remaining), using the chance of winning the tournament outright is a fair estimate of one's share of the prize pool, particularly for good players who will attempt to maximize their equity appropriately at money steps.
Using these two assumptions, we can construct a model.

Consider a tournament with X players of equal skill who begin with equal stacks. We will consider a single player. Let E be his chance of winning the tournament

outright, C be his chance of doubling his stack before busting, and N be the number of times he must double his stack in order to win.
Then we have the simple relationship E = CN.

Now if we have any two of these quantities we can calculate the third. Let's consider the situation at the beginning of the tournament. Since all the players are equally skilled, we know that our player has an equity E = 1/X.
Now we also know that in order to win the tournament, our player must increase his stack from its starting value to X times that amount. The number of doubles that this requires can be expressed as:
X = 2N

So for a 2 player tournament, N would obviously be 1, as one double would be required to win, while for a 128 player tournament, N is 7. More generally:
N = log2 X
Substituting:
1/X = Clog2 X
log2 1/X = (log2 X) log2 C
log2 C = -1
C = .5

Of course, this is what we would expect for a tournament where all players are equally skilled; that each player would have a 50% chance of doubling before busting from any stack size. It is easily verified by plugging in other values of N (representing different stack sizes) that chip values are exactly linear for players of equal skill.

However, this methodology now generalizes to situations where the players are of unequal skill. For example, consider a hypothetical player A playing in a 100 player tournament where he has EV of 1 buyin net (2 buyins per tournament gross). We can calculate his C by plugging in his initial values:

$E = CN$

$2/100 = C\log_2 100$

$.02 = C$

6.643856

$C = .5550$

So A has a chance of doubling his current stack before busting of 55.5%.
Now we can calculate his chance of winning the tournament from any stack size S (where S is the number of starting stacks he has) by plugging into the equity formula E = CN. N, of course, is the number of doubles it will take to reach 100.

$E = C\log_2 (100/S)$

So for a stack size of 2 (double the initial stack), A has equity of $E = C\log_2 50$, or .0360. Note that this value is more than the *a priori* value of the chips (which would of course be .02), but less than double A's chance of winning the tournament from the start.

What this model does, in effect, is provide us a means for evaluating marginal tournament equity decisions *including the skill of the player making the decisions.* The qualitative judgment of "I have an edge over the field, so I should protect my chips" is quantified into an adjusted equity for any stack size that utilizes the assumptions that underlie the model. To see how this model might work in practice, consider the following marginal situation from a no-limit holdem tournament.

The blinds are 75-150. Player B, with a stack of 3000 (250 players played with starting stacks of 1500), raises to 450 on the button with QsTs. The small blind folds and the big blind calls. The flop now comes Ks 8c 2s. The blind checks and player B bets 500. The blind now check-raises all-in (and has B covered). B estimates his equity at 36% and is faced with the decision of calling 2050 to win 4025.

Now an analysis based purely on tournament chip values would indicate that calling is clear—.36 * (6075)—2050 = 137 chips, for nearly a tenth of a buyin worth of profit. But let's assume that Player B had an edge of Ω of a buyin over the field at the beginning of the tournament. Calculating C for Player B:

1.75/250 = Clog2 250

C = .5364

Now we can calculate the equity of the three scenarios:

If B calls and wins, he will have 6075 chips, or 4.05 starting stacks.

E = .5364log2(250/4.05)

= .024603 buyins.

If B calls and loses, he will have 0 chips and have 0 equity.

If B folds, he will have 2050 chips, or 1.36667 starting stacks.

E = .5364log2(250/1.36667)

= .009269 buyins.

So B's equity from calling is .024603 * .36, or .008857 buyins. Comparing this to B's equity from folding, we find that the call is actually *incorrect* given B's skill edge over the field.

Another result that can be easily derived from the Theory of Doubling Up include the "coinflip" problem—what chance of winning do you need to accept a coinflip for all your chips on the first hand? Since E0 = CN at the start of the tournament, if you call and double up, your equity

becomes E1 = CN-1.
If you take a coinflip with chance of winning W, then your equity becomes WE1.
If you decline then your equity is E0. Setting these things equal, we get:
CN = WCN-1, or W = C.

So you are indifferent to accepting a coinflip if your chance of winning the coinflip is equal to C. Yet consider that many players have a stated preference that they would decline a QQ vs AK (57-43) confrontation in the early tournament. By applying the Theory of Doubling Up, we can find out something about what these players believe their equity in, for example, a 250 player tournament to be.
E = CN = (.57)8 = .01114, or about 2.85 buyins per tournament. In order for it to be to correct to decline a 57-43 confrontation with no dead money in the pot, one has to have nearly three times the average equity in the tournament. Our observations lead us to believe that having such a win rate in a typical tournament is extremely unlikely.

Another interesting effect of the Theory of Doubling Up is what it indicates about players with negative expectation. Of course our readers are all winning players, but nevertheless, the mean result of all players in a tournament is to lose the entry fee, so someone must be losing.

The Theory of Doubling Up, then, indicates that losing players should be willing to commit their chips even in marginally bad situations. This is a result of the idea that losing players should encourage variance, as it is their best chance to win. The more that other players get a chance to apply their skill, the worse off the losing player will be. Hence they should be

willing to commit all their chips in zero-EV situations or even slightly bad ones.
Summarizing:

At the beginning of the tournament, each player has a chance of doubling up C, which is related to his *a priori* overall equity E0 in the tournament by the following equation:
E0 = Clog2 P d
where P is the number of players in the tournament.
Then from any stack size S, C needs to double his stack a certain number of times in order to have all the chips.
N = log2 (P/S)
And finally, for any stack size,
E = CN.
These equations can be utilized to estimate tournament equity on a skill-adjusted basis.